Ponyville Statistical Trivia

Volume One: Population

- by dziadek1990

http://dziadek1990.deviantart.com

How many are there citizens of Ponyville?
This question sprouted my interest, but I didn't want to half-ass the calculations, so I used all my skills to get the most accurate answer that could be estimated by a person with my level of education.

I used two different methods to estimate it:

1. "Mark and recapture" – a method commonly used by ecologists to estimate a given population's size by analyzing 2 groups of members taken from that population;
2. Calculating, the number of ponies that would be required to drink all cider in the 2x15 "Super Speedy Cider Squeezy 6000" episode based on references from the show, assuming each one took only a mug.

So… here we go!

1. "Mark and recapture"

In short: I analyzed screenshots of 9 different crowds of characters, counted their sizes, and how many ponies did each of them share – and then used a following formula:

$$S = \frac{A \cdot B}{C};$$

S – Size of entire population;
A – Size of the first crowd of ponies (the 'marked' group);
B – Size of the second crowd of ponies (the 'recaptured' group);
C – Number of ponies both crowds share;

Ecologists would catch some animals from a given population, mark them, release them, capture more animals after a while, and then count how many of animals they captured had the marks on them.

If, say, A = 20, B = 30 and C = 5, it would mean that after marking 20 animals, we found 5 of them in a group of 30 animals we caught later.

The chance of catching those animals again was equal to A divided by S. If half of animals in population were marked, then we would have 50% chance of catching a single marked animal – assuming random distribution.

So we have 20 marked ponies, and found 5 of them in another group of 30 ponies – so those 20 ponies constitute as $5/30^{th}$ of the entire population, or $1/6^{th}$.

6 times 20 is 120 – the estimated number of ponies in a population from which the both groups were drawn.

After clearing that up I will proceed to showing the numbers I got from comparing 9 different crowds of characters, 36 pairs of crowds total.

Returning to my calculations (I also counted Spike – yes, he is not a pony, but we can all agree that he is a citizen of Ponyville, right?).

First, I'll list the crowds:

#1 crowd, 30 characters:

#2 crowd, 42 characters:

Pictures don't fit well on the pages, so I am forced to leave gaps like this.
Suggestions what should I fill them with? More ponies? Unrelated mini-trivia? Boobs?
You can tell me when you finish reading.

#3 crowd, 23 characters:

#4 crowd, 14 characters:

#5 crowd, 16 characters:

#6 crowd, 59 characters:

#7 crowd, 29 characters:

#8 crowd, 27 characters:

#9 crowd, 16 characters:

Most of the time you can't see the ponies' cutie marks, so I focused only on their colors and shapes of manes to identify them – if I saw only the tip of the mane, tail, or just the legs, I ignored the pony, because the rest of the body could be of any shape or color.

There are many identical characters so you can't be sure if two same-looking ponies are the same character, so the numbers of the ponies I counted that were present in both crowds actually represent the **maximal** number of ponies those crowds **could** share – because just as well none of the same-looking characters could be the same ponies.

In order to estimate the population size S I divided the earlier mentioned A and B by this shared number C. Since I counted the maximal number of ponies present in both crowds, the numbers estimating the population in this case represent the minimal population – its lower limit.

C could have a value shown in the table below, or a lower one – so S, since it is inversely proportional to C, could have a value shown below, or a higher one.

Sizes of groups:

Group	Size
1	30
2	42
3	23
4	14
5	16
6	59
7	29
8	27
9	16

Average: 28;

Number of ponies they share (max):

Shared	1	2	3	4	5	6	7	8	9
1		1	1	3	2	4	4	3	2
2			10	2	2	10	1	1	1
3				2	2	6	1	1	2
4					2	5	7	6	6
5						6	4	5	3
6							5	4	3
7								7	7
8									6
9									

Average: 4;

Estimated population for any given pair (min):

Estimated	1	2	3	4	5	6	7	8	9
1		1260	690	140	240	443	218	270	240
2			97	294	336	248	1218	1134	672
3				161	184	226	667	621	184
4					112	165	58	63	37
5						157	116	86	85
6							342	398	315
7								112	66
8									72
9									

Average: 326;

Answer: The average lower limit of Ponyville's population is 326 ponies – but we have no idea exactly HOW much bigger the actual population is. This method proved not very helpful.

(But I still had fun while playing with it – otherwise how the buck would I have the patience to do all this stuff?)

Now the second method.

2. One mug per pony – no more, no less.

Twilight Sparkle: “I spy… an 8 foot candy cane!”

Thanks, Twi.

Assuming all foals have heads of the same size…

Average barrel diameter = biggest + smallest divided by 2 = 59 cm (1’11”).
Barrel radius r = diameter divided by 2 = 30 cm (1’0”).
Barrel height h = 73 cm (2’5”).

This way I simplified the barrel's shape into a cylinder.
Knowing the formula for a cylinder's volume…

$$Barrel's\ volume\ v = \pi \cdot r^2 \cdot h = 206\ liters\ (54\ gallons);$$

Number of barrels: 24.
Total volume: 4,953 liters (1,307 gallons).
We know that it is not enough for gathered ponies.
◄

Number of barrels: 33.
(3 barrels on top, each level below has 1 barrel more than the one above).
Total volume: 6,810 liters (1,797 gallons).
We know that this is enough for gathered ponies.
▼

Returning to the previous picture… let's look at that mug of cider.

After a modification - also a cylinder.
Mug volume: 6.9 liters (1.8 gallons).

24 barrels hold 718 mugs of cider – not enough for the thirsty citizens.
33 barrels hold 987 mugs of cider – enough.

One mug per pony.

(I know Pinkie took a lot of mugs by herself, but this number is small compared to the overall quantity of cider. The assumption that each pony took a singular mug still remains).

Further estimation of population

I will assume citizens of Ponyville live as long as humans – otherwise, given how much time has passed since the first episode, the CMC would be teenagers/ young adults at this point.

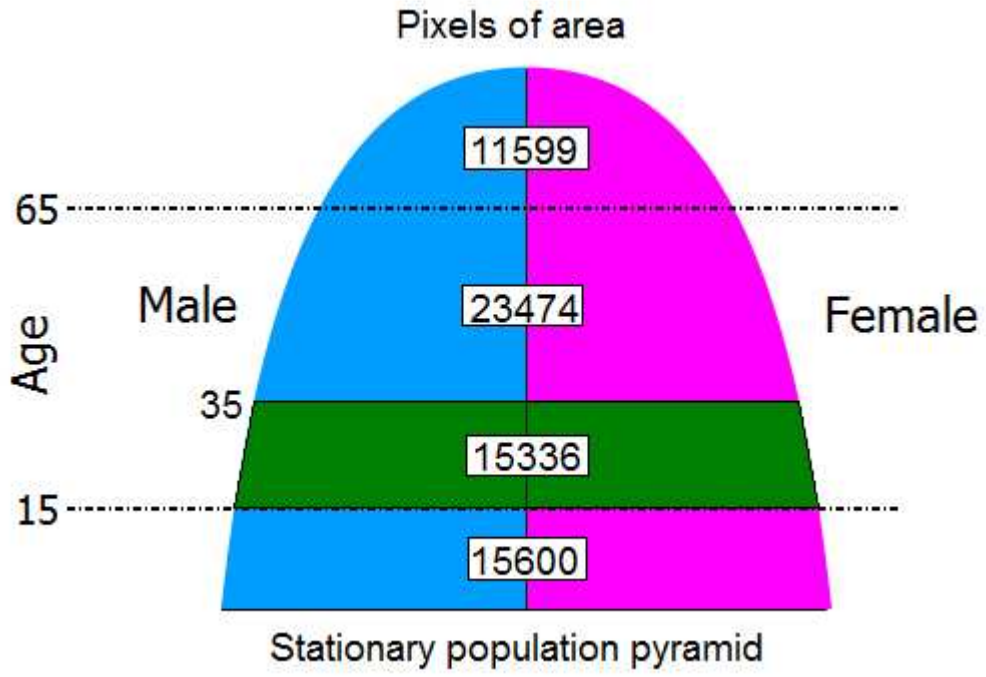
Foals and the elderly were not present at the cider sales – apparently the age range was somewhere like 15 – 35. I calculated their numbers based on an age distribution of a Ponyville's population.

There are four main types of population pyramids:

- Stable – Unchanging age distribution. Average mortality and fertility.
- Stationary – Unchanging age distribution. Low mortality and fertility.
- Expansive – Rapid population growth. High fertility and mortality. Often seen in poor countries with little-to-no birth control and poor health care.
- Constrictive – Low death rate and low fertility rate. Population of the oldies.

I assumed Ponyville represents Stationary Population Pyramid (being the good, pure and safe Utopia it is).

The surface area in this chart represents the number of citizens within a given age range.



(No, I didn't count each individual pixel – just calculated the approximate area of each sector and didn't round the estimation. I am random when it comes to rounding; sometimes I do, sometimes – I don't)

We can ignore the gender distribution – the age is all that should interest us at the moment.

Even though it seems majority of the shown ponies are between 15 and 35 old, let's say that the 'camera' just had a tendency of recording the areas popular amongst ponies of that age; the areas with ponies of other ages were shown less frequently.

Reminder:
Young population exists only in the high-mortality-and-fertility countries/towns, and in artificial populations, like, say resorts where young people are allowed – and then kicked out once they reach a certain age. I think we both agree that Ponyville is neither of those and it has a Stationary Population Pyramid, even though it seems to the contrary.

To avoid confusion, I am going to exclude an assumption that, despite their looks, a lot of ponies are actually above 35 or below 15 years old.

Returning to the chart. 15–35 age range constitutes as 23% of the surface area, so the characters within that range constitute as the same fraction of total population.
From earlier calculations we know that there are 718 – 987 ponies in that age range.

If 718 is 23%, the 100% would be 3,122.
If 987 is 23%, then the 100% would be 4,291.

It is not impossible that some 15–35-year-olds were not present, but assuming all of them were there and got to drink the cider, then the population would fall somewhere between 3,122 and 4,291. Otherwise, it could be even bigger than that upper limit.

Final estimation: Ponyville has between 3,100 and 4,300 citizens.

That's it for now. I decided not to calculate every single statistical trivia in one go, in order to go for quality over quantity.

Asking people who aren't math geeks:
Were some sections of the volume too confusing? Too many or too little formulas compared to the rest of the text? Did I make it entertaining enough?

Too other math geeks:
Some glaring math errors I didn't notice?

Did I… *GASP*… FORGET TO CARRY THE TWO???

To writers:
Please, use your writer-mojo (if that is even possible in this case)and tell me what were the flaws in my presentation and where I've done well. Suggestions how to make it more entertaining? (Or less boring, if somehow I managed to screw everything up badly without even realizing it).

Being just a beginner writer, I have a lot to learn in this case.

Special thanks to BryanD for making me aware of the "Mark and recapture" method and, of course, to all wonderful people who were and are working on the show. Never had so much fun in my life since I joined the community.

I will use the rest of suggestions given to me in the next analysis.

So… see ya later!

Your friendly neighborhood geek, dziadek1990. ☺

April 3rd, year 2012.